

Criticizing Bell: Local Realism and Quantum Physics Reconciled

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A typical sample of Bell's inequality is proved to require, besides the standard assumptions on realism and locality, the adoption of a metatheoretical classical principle for interpreting quantum laws. A new principle is proposed which is consistent with the operational philosophy of quantum physics; it is then shown that, whenever the latter principle is adopted in place of the former, realism (here intended in a purely semantical sense) and locality do not imply Bell's inequality in the form considered here, but a new inequality which is not violated in quantum physics. Thus an interpretation of quantum physics that is (semantically) realistic and local is suggested, which eliminates a number of seeming paradoxes.

1. INTRODUCTION

Bell's inequality (BI) was formulated in 1964 (Bell, 1964). Since then a number of variants have been proposed by several authors [a survey on this subject can be found in Selleri (1988a)] and it has become a relevant topic in the studies on the foundations of quantum physics (QP).

It is usually maintained [see again Selleri (1988a)] that BI can be deduced by using only the following (seemingly weak) assumptions.

R. The results of all conceivable measurements on a sample of a given physical system are simultaneously pre-fixed (even in the case of incompatible observables).

LOC. A measurement made on a system 1 does not modify the pre-fixed values of the observables of another system 2, if 1 and 2 are sufficiently far apart (even if 1 and 2 have interacted in the past).

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Regarding R, it must be noted that it is sometimes identified with the *minimal realism* (MR) which is implicit in the reasoning of most physicists, whatever their claimed worldviews may be, that is, with the generic belief that some kind of “external reality” exists which is independent of the observer and is explored by our empirical and conceptual instruments (notice that MR must not be confused with *ontological realism*, which is a much more binding philosophical position). This identification is incorrect, since R is compatible not only with MR, but also with different epistemological attitudes. But it is important to observe that, in any case, renouncing R raises a lot of epistemological problems (in particular, it opens the way to the introduction of some forms of subjectivism in physics).

Regarding LOC, I limit myself here to note that it cannot be given up without renouncing the principle of separability in QP [see in particular D’Espagnat (1976)].

However, it is well known that BI is violated in QP, and this violation is confirmed by a number of experimental results. This, R, or LOC, or both seem to be falsified by QP (with regard to R, this result is obviously consistent with the philosophical attitude adopted by the Copenhagen school). This explains the uneasiness expressed by many authors and induces a number of exotic interpretations of QP that can be considered seriously, in my opinion, only when their devastating epistemological implications are underestimated.

It is therefore relevant to inquire whether R and LOC can be reconciled with QP. The proof that this is possible is the present paper’s object.

My reasoning will be worked out in the framework of an epistemological position defended by myself in some recent papers (Garola, 1991; 1992a—c) that will be called here *semantical realism* (SR). Therefore, let me recall that, basically, SR assumes that every interpreted sentence in the language of a physical theory has a truth value, thus accepting and reformulating R. In addition SR also assumes that all formal properties of the concept of truth are established by the standard Tarski theory of truth, but truth (which is carefully distinguished from epistemic accessibility, or testability) is conceived as a purely semantical notion, and any ontological commitment about the existence of physical entities is avoided (yet not prohibited in principle; in this sense one can say that SR is *ontologically neutral*).

In the framework of SR a preliminary analysis is made in Section 2 of the assumptions that are actually needed in the standard proofs of BI. Then, a new metatheoretical principle (MGP) is stated in Section 3 which generalizes a classical principle that is adopted (usually implicitly) when characterizing the truth mode of physical laws (both in quantum and in classical physics). Finally, it is proved in Section 4 that accepting MGP invalidates BI without implying any renouncement of R and/or LOC, so that the violation of BI in

QP does not oblige us to give up these basic premises. Some comments on this result are added in Section 5.

2. BELL's INEQUALITY

I will take into account here only the elementary discussion of BI supplied in Sakurai's textbook (Sakurai, 1985); it is then an exercise to show that the arguments in this paper also apply to the alternative forms of BI that can be found in the literature².

The analysis will be carried out by using the following definitions and symbols.

First, for every physical system, let \mathcal{P} be the set of all *testable physical properties*, the set of all properties that can be tested to be true or false on any individual sample of the system (here called *physical object*) by means of suitable dichotomic measuring apparatuses. Second, let Ψ be a set of *well-formed formulas* (wffs), recursively constructed as follows: (i) let \mathcal{A} be an alphabet containing a set of symbols $\Psi_T = \{E, E_0, E_1, \dots\}$ denoting testable properties in \mathcal{P} and the standard connectives \neg, \wedge, \vee of classical logic; (ii) let every symbol in Ψ_T be a (atomic) wff, and for every wff A, A_1, A_2 , let $\neg A, A_1 \wedge A_2, A_1 \vee A_2$ be (molecular) wffs. In addition, let an *extension* $\rho_i(A)$ be associated to every $A \in \Psi$ in every laboratory i [space-time domain, see Garola (1991)], where a set D_i of physical objects is prepared, by means of the following recursive rules: (i) for every $E \in \Psi_T$, $\rho_i(E)$ is the set of all physical objects in i that have the property denoted by E , that is, the set of all physical objects in i that would yield the answer yes if tested with an ideal dichotomic measuring apparatus associated to E [note that this definition does not require the actual knowledge of $\rho_i(E)$, which is usually impossible in QP even if one knows the state of every physical object in D_i ; see Garola (1992a)]; (ii) for every $A \in \Psi$, $\rho_i(\neg A) = D_i \setminus \rho_i(A)$ [hence, whenever $A \in \Psi_T$, $\rho_i(\neg A)$ is the set of all physical objects in i that would yield the answer no if tested with an ideal dichotomic measuring apparatus associated to A]; (iii) for every $A_1, A_2 \in \Psi$, $\rho_i(A_1 \wedge A_2) = \rho_i(A_1) \cap \rho_i(A_2)$ and $\rho_i(A_1 \vee A_2) = \rho_i(A_1) \cup \rho_i(A_2)$. Finally, assume that, whenever one considers a given physical object, every $A \in \Psi$ has a truth value (true or false) according to the standard

²For instance, in the Wigner proof of BI (Wigner, 1970), which is an ancestor of the Sakurai proof, MGP in Section 3 invalidates Wigner's statement that " $(\sigma_1, \sigma_2, \sigma_3; \tau_1, \tau_2, \tau_3) = 0$ unless $\tau_1 = -\sigma_1, \tau_2 = -\sigma_2, \tau_3 = -\sigma_3$." It must be noted that in other cases the name BI is attributed to an inequality of the form $\Delta \leq 2$, where Δ depends on a suitably defined *correlation function* (Selleri, 1988a,b); in this case MGP does not lead one to question BI directly, but rather to invalidate the identification between the correlation function that appears in Δ and the correlation function introduced in QP.

Tarskian truth theory [i.e., A is true if the physical object belongs to $\rho_i(A)$, false otherwise]³.

It is important to understand clearly that $\rho_i(A_1 \wedge A_2)$ and $\rho_i(A_1 \vee A_2)$ usually do not coincide with extensions of symbols denoting testable properties, while $\rho_i(\neg A)$ does, because of the above interpretation. Indeed it is expedient to follow here the same procedures established in Garola (1991), where quantum logic is obtained by selecting suitable subsets of (atomic and molecular) wffs that are testable according to QP in the set of all wffs of a classical language with testable atomic wffs.

Bearing in mind the aforesaid procedures, one is led to define first a preorder on Ψ (denoted here by the same symbol \subseteq that denotes set inclusion, no confusion being possible) by setting:

$$\text{for every } A_1, A_2 \in \Psi, \quad A_1 \subseteq A_2 \text{ iff for every laboratory } i, \quad \rho_i(A_1) \subseteq \rho_i(A_2)$$

and to introduce the *completion by cuts* (Garola, 1985) $(\overline{\Psi}_T, \subseteq)$ of (Ψ_T, \subseteq) , that possibly adds *theoretical* properties, endowed with suitable conventional extensions, to the testable properties in Ψ_T . Then, the lattice operations $^\perp$, \cap , \cup defined on $(\overline{\Psi}_T, \subseteq)$ can be interpreted as follows (here \equiv is the equivalence relation induced by \subseteq , hence it denotes identity of extensions in every laboratory): (i) for every $E \in \overline{\Psi}_T$, $E^\perp \in \overline{\Psi}_T$ denotes a property whose extension coincides with the extension of $\neg E$ in every laboratory, that is, such that $E^\perp \equiv \neg E$; (ii) for every $E_1, E_2 \in \overline{\Psi}_T$, $E_1 \cap E_2$ denotes (up to \equiv) the greatest property which is smaller (according to \subseteq) than E_1 and E_2 ; $E_1 \cup E_2$ denotes the smallest property which is greater than E_1 and E_2 .

It is well known that $(\Psi/\equiv, \subseteq)$ (where \subseteq now denotes the order in Ψ/\equiv induced by the preorder \subseteq defined on Ψ) is a complete Boolean lattice, while $(\overline{\Psi}_T/\equiv, \subseteq)$ [which can be identified, in particular, with the Jauch–Piron lattice of propositions; see Garola (1991)] is a complete, orthocomplemented, weakly modular, atomic lattice which satisfies the covering law. Furthermore, in the Hilbert space quantum theory (HSQT) $(\overline{\Psi}_T/\equiv, \subseteq)$ can be identified with the poset of the orthogonal projections in the Hilbert space of the system; one briefly says that every property is represented by an orthogonal projection. Then compatible (i.e., conjointly testable) properties are represented by commuting projections (and conversely), and it is important to note that, whenever E_1 and E_2 are compatible properties, one can assume that $E_1 \cap E_2 \equiv E_1 \wedge E_2$.

³The assumption that all symbols denoting testable properties are wffs and the attribution of truth values to wffs of the kind constructed here are somewhat anomalous with respect to standard procedures in classical logic, since no symbol denoting individual constants or variables appears explicitly in the wffs of Ψ . I made this choice since it favors brevity and understandability in this paper.

Consider now a system of two spin-1/2 particles, say 1 and 2, and let \mathcal{H}_S be the 4-dimensional spin space of the system. Let U_{1+} denote the property “particle 1 has spin up in the \mathbf{u} direction”; then the further symbols U_{1-} , U_{2+} , U_{2-} , V_{1+} , V_{1-} , V_{2+} , V_{2-} , W_{1+} , W_{1-} , W_{2+} , W_{2-} , ... have an obvious meaning. All these properties are represented in \mathcal{H}_S by two-dimensional projections. Furthermore, let S_0 and E_0 respectively denote the singlet state of the system and the physical property (“the system has total spin zero”) associated to it [the state S_0 can be represented in HSQT by the same one-dimensional projection that represents E_0 ; nevertheless S_0 and E_0 cannot be identified, since their extensions are generally different; see Garola (1992a)].

By using these definitions and symbols, and assuming that \mathbf{u} , \mathbf{v} , \mathbf{w} , ... are neither parallel nor antiparallel, one gets

$$E_0 \equiv ((U_{1+} \cap U_{2-}) \cup (U_{1-} \cap U_{2+})) \cap ((V_{1+} \cap V_{2-}) \cup (V_{1-} \cap V_{2+})) \cap ((W_{1+} \cap W_{2-}) \cup (W_{1-} \cap W_{2+})) \cap \dots$$

This equivalence can be easily justified by considering the projections which represent the properties that appear in it; it is also easy to see that it can be simplified as follows:

$$E_0 \equiv ((U_{1+} \cap U_{2-}) \cup (U_{1-} \cap U_{2+})) \cap ((V_{1+} \cap V_{2-}) \cup (V_{1-} \cap V_{2+}))$$

Since every property with index 1 in the second term of the above equivalence is compatible with every property with index 2, one also gets

$$E_0 \equiv ((U_{1+} \wedge U_{2-}) \cup (U_{1-} \wedge U_{2+})) \cap ((V_{1+} \wedge V_{2-}) \cup (V_{1-} \wedge V_{2+})) \cap ((W_{1+} \wedge W_{2-}) \cup (W_{1-} \wedge W_{2+})) \cap \dots$$

Consider a set of samples of the system (physical objects) that are in the state S_0 (at a given time t) in the laboratory i ; then, one expects that, for every sample, E_0 must be true (at time t) in i . This statement constitutes an instance of *physical law* (expressed in natural language), in which the knowledge of the state of a physical object allows one to predict that the object has a given physical property (Garola, 1992a). By using this law, the following statement P can be proven.⁴

⁴Statement P can be proven as follows. Consider a sample of the system in i such that E_0 is true; one obtains at once that $Q_u = (U_{1+} \cap U_{2-}) \cup (U_{1-} \cap U_{2+})$ must also be true. Then, consider the projections that represent U_{1+} , U_{1-} , U_{2+} , U_{2-} ; it follows $U_{1-} \equiv U_{1+}^\perp$, $U_{2-} \equiv U_{2+}^\perp$. Therefore, $(U_{1-} \cap U_{2+})^\perp \equiv U_{1+} \cup U_{2-}$; hence $U_{1+} \cap U_{2-} \subseteq (U_{1-} \cap U_{2+})^\perp$. By using the property of weak modularity in the lattice ($\Psi \neq \equiv, \subseteq$) one gets $U_{1+} \cap U_{2-} \equiv (U_{1-} \cap U_{2+})^\perp \cap ((U_{1+} \cap U_{2-}) \cup (U_{1-} \cap U_{2+})) \equiv (U_{1+} \cup U_{2-}) \cap Q_u$. Let the given sample of the system be such that U_{1+} is also true. Then $U_{1+} \cup U_{2-}$ is true since U_{1+} is true, Q_u is true as we have seen above, hence $U_{1+} \cap U_{2-}$ is true, which implies, since now $U_{1+} \cap U_{2-} \equiv U_{1+} \wedge U_{2-}$, that U_{2-} is true. The rest of the proof is straightforward.

P. For every sample of the physical system in the state S_0 , U_{1+} (U_{1-}) is true iff U_{2-} (U_{2+}) is true, and V_{1+} (V_{1-}) is true iff V_{2-} (V_{2+}) is true, and W_{1+} (W_{1-}) is true iff W_{2-} (W_{2+}) is true, and ...

Statement P is basic in every proof of BI, as will be presently shown in the particular case of the Sakurai proof. To this end, it is convenient to introduce the following further definitions and symbols. First, for every testable property E and state S let $p(E/S)$ denote the conditional probability (which can be evaluated in QP) that a physical object in the state S has the property E . Second, in every laboratory i , let the extension $\rho_i(S)$ of the state S be the set of all physical objects in i that are in the state S . Third, let $n(\sigma)$ denote the number of elements in the finite set σ , and in every laboratory i and for every $A \in \Psi$, put $f_i(A/S) = n(\rho_i(A) \cap \rho_i(S))/n(\rho_i(S))$ [whenever A is equivalent to a testable property E , $f_i(A/S)$ approaches $p(E/S)$ in every laboratory i if the number of physical objects is sufficiently large, hence QP allows one to evaluate $f_i(A/S)$; on the contrary, no evaluation is supplied by QP whenever A is not equivalent to a testable property, and $f_i(A/S)$ might vary from one laboratory to another].

Let us come to the proof of BI. This relies on the following assumptions R_S and LOC_S , which restate assumptions R and LOC, respectively, in our present SR context (note that R_S is the basic assumption of SR).

R_S . Every testable physical property has a defined (though possibly unknown) truth value, independently of the act of observation.

LOC_S . Whenever particles 1 and 2 are spatially well separated, the truth value of every testable property of particle 1 is not influenced by any measurement carried out on particle 2.

If R_S and LOC_S are accepted, one can write in the laboratory i

$$\begin{aligned} p(U_{1+} \cap V_{2+}/S_0) &\cong f_i(U_{1+} \wedge V_{2+}/S_0) \\ &= f_i(U_{1+} \wedge V_{2+} \wedge W_{1+}/S_0) + f_i(U_{1+} \wedge V_{2+} \wedge W_{1+}^\perp/S_0) \\ p(U_{1+} \cap W_{2+}/S_0) &\cong f_i(U_{1+} \wedge W_{2+}/S_0) \\ &= f_i(U_{1+} \wedge V_{2+} \wedge W_{2+}/S_0) + f_i(U_{1+} \wedge V_{2+}^\perp \wedge W_{2+}/S_0) \\ p(V_{2+} \cap W_{1+}/S_0) &\cong f_i(V_{2+} \wedge W_{1+}/S_0) \\ &= f_i(U_{1+} \wedge V_{2+} \wedge W_{1+}/S_0) + f_i(U_{1+}^\perp \wedge V_{2+} \wedge W_{1+}/S_0) \end{aligned}$$

It has already been observed in footnote 4 that $U_{1+}^\perp \equiv U_{1-}$; analogously, $V_{2+}^\perp \equiv V_{2-}$ and $W_{1+}^\perp \equiv W_{1-}$. Furthermore, statement P implies the following equation.

$$\mathbf{F.} \quad f_i(U_{1+} \wedge W_{2+}/S_0) = f_i(U_{1+} \wedge W_{1-}/S_0).$$

Hence one gets

$$p(U_{1+} \cap V_{2+}/S_0) \cong f_i(U_{1+} \wedge V_{2+} \wedge W_{1+}/S_0) + f_i(U_{1+} \wedge V_{2+} \wedge W_{1-}/S_0)$$

$$p(U_{1+} \cap W_{2+}/S_0) \cong f_i(U_{1+} \wedge V_{2+} \wedge W_{1-}/S_0) + f_i(U_{1+} \wedge V_{2-} \wedge W_{1-}/S_0)$$

$$p(V_{2+} \cap W_{1+}/S_0) \cong f_i(U_{1+} \wedge V_{2+} \wedge W_{1+}/S_0) + f_i(U_{1-} \wedge V_{2+} \wedge W_{1+}/S_0)$$

By comparing these equations one finally obtains *Bell's inequality*.

$$\mathbf{BI:} \quad p(U_{1+} \cap V_{2+}/S_0) \leq p(U_{1+} \cap W_{2+}/S_0) + p(V_{2+} \cap W_{1+}/S_0).$$

The above proof of BI makes explicit that BI does not depend on R_S and LOC_S only, but also on P. This last dependence is crucial here. Indeed, the violation of BI in QP has induced most authors to maintain that QP is incompatible with R and/or LOC, which is highly problematic, as I have already pointed out in the Introduction. But looking at the above proof, one may suspect now that QP only conflicts with the implicit assumption that P must be true in (almost⁵) every laboratory (hence in *i*). Indeed, I will maintain in the next section that this assumption holds whenever one adopts a classical conception of the laws of physics (which must not be confused with the adoption of classical logic, or of classical models), while P is not necessarily true in every laboratory whenever one adopts a generalization of the classical conception that is consistent with the operational philosophy of QP.

3. THE METATHEORETICAL GENERALIZED PRINCIPLE

Let us recall preliminarily that, according to a standard epistemological conception [*received viewpoint*; see Carnap (1966)] one must distinguish between *theoretical physical laws* and *empirical laws*. The former are sentences of a very general language L^* (Garola, 1992a) which contain *theoretical terms* and cannot be directly tested. The latter are deduced from the former, via *correspondence rules*, and belong to an observative sublanguage of L^* . In the elementary language constructed above a theoretical or empirical law A is a molecular sentence composed of subsentences A_1, A_2, \dots whose truth values can be determined empirically (notice that it is not requested at this stage that the truth values of A_1, A_2, \dots can be determined conjointly; this point becomes relevant in QP, where the mere testability of the subsentences does not imply that they are conjointly testable).

Then, the following principle, which characterizes the truth mode of physical laws (Garola, 1991), formalizes a metalinguistic assumption that is

⁵The word *almost* has a technical meaning that has been discussed in a previous paper (Garola, 1991). I will not insist on this point here, for the sake of simplicity.

usually made implicitly when using classical or quantum laws in order to predict properties of physical objects.

MCP. (Metatheoretical classical principle). A physical law is expressed by a sentence A which is true in every laboratory.

The above MCP implies that in every laboratory the truth values of all component subsentences of a physical law A must be such as to make A true. It is then easy to show that F follows from P because of MCP. Indeed, for every physical object such that U_{1+} and W_{2+} are conjointly true, also U_{1+} and W_{1-} must be conjointly true, otherwise P would be false in i , contrary to MCP; the same argument shows that, if one assumes that U_{1+} and W_{1-} are conjointly true (even if this cannot be verified in QP, since U_{1+} and W_{1-} are not compatible), then U_{1+} and W_{2+} must be conjointly true. Hence F immediately follows.

Now, it is apparent that MCP is not consistent with the operational philosophy of QP. In fact, from an operational viewpoint, one can legitimately assert that a sentence A expresses a physical law if it is always true in its "domain of testability," i.e., roughly speaking, in every physical situation that can be concretely observed, while it seems arbitrary to require that A must remain true, as MCP does, when a physical situation is hypothesized which, in principle, cannot be observed. This suggests that we generalize MCP so as to obtain the following principle, which characterizes in a new way [even with respect to a previous proposal by the author, which essentially rephrased MCP; see Garola (1991)] the truth mode of physical laws.

MGP. (Metatheoretical generalized principle). A physical law is expressed by a sentence A which is true in every laboratory where conjointly empirically determined (or determinable) truth values are attributed to some of its testable component subsentences.

MGP obviously implies that a physical law A imposes some relationships among the truth values of its testable subformulas A_1, A_2, \dots whenever the truth values of some of them are conjointly determined by means of suitable tests; in this sense, it is analogous to MCP. But A does not impose now any relationship among the truth values of A_1, A_2, \dots whenever truth values are assigned to some of them which are not conjointly accessible (one can say that a physicist cannot engage on the truth value that A has in this case). Therefore, the difference between MGP and MCP is irrelevant in CP, where the truth values of all sentences are empirically determinable conjointly, while it is basic in QP, where MGP is consistent with the operational philosophy of QP.

4. THE CRITIQUE OF BELL'S INEQUALITY

Let us come now to the critique to the proof of BI that has been provided in Section 2.

Whenever MGP is adopted in place of MCP, F cannot be inferred from P. Indeed, if the system is in the state S_0 , one can still assert that, for every physical object, such that U_{1+} and W_{2+} (compatible properties) are conjointly true, also U_{1+} and W_{1-} are conjointly true (even if this latter statement cannot be tested directly), since MGP states that P must be true in this case. But one is not authorized to say that, for every physical object such that U_{1+} and W_{1-} (noncompatible properties) are conjointly true, also U_{1+} and W_{2+} are conjointly true, since MGP does not give any guarantee that P must be true in this case. Thus one gets the following weaker statement in place of F.

$$\mathbf{F}_Q. \quad f_i(U_{1+} \wedge W_{2+}/S_0) \leq f_i(U_{1+} \wedge W_{1-}/S_0).$$

This result is important. Indeed, it shows that the adoption of MGP implies a breakdown in the reasoning that leads to BI; thus BI no longer follows from R_S and LOC_S . In place of BI, R_S and LOC_S impose a weaker, harmless inequality, that can be found by using the procedures in Section 2 with F_Q in place of F. One gets:

$$\begin{aligned} p(U_{1+} \cap V_{2+}/S_0) &\cong f_i(U_{1+} \wedge V_{2+} \wedge W_{1+}/S_0) + f_i(U_{1+} \wedge V_{2+} \wedge W_{1-}/S_0) \\ p(U_{1+} \cap W_{2+}/S_0) &\leq f_i(U_{1+} \wedge W_{1-}/S_0) \\ &= f_i(U_{1+} \wedge V_{2+} \wedge W_{1-}/S_0) + f_i(U_{1+} \wedge V_{2-} \wedge W_{1-}/S_0). \\ p(V_{2+} \cap W_{1+}/S_0) &\cong f_i(U_{1+} \wedge V_{2+} \wedge W_{1+}/S_0) + f_i(U_{1-} \wedge V_{2+} \wedge W_{1+}/S_0) \end{aligned}$$

Hence one can write the following *weakened Bell's inequality* in every laboratory i .

$$\mathbf{WBI.} \quad p(U_{1+} \cap V_{2+}/S_0) \leq f_i(U_{1+} \wedge W_{1-}/S_0) + p(V_{2+} \cap W_{1+}/S_0).$$

It is apparent that WBI (which is not a physical law in the sense specified above) cannot be contradicted by any experimental result, since U_{1+} and W_{1+} are not compatible.

5. CONCLUSIONS AND COMMENTS

The above discussion shows that the violation of BI in QP does not invalidate R_S and/or LOC_S , hence R and/or LOC, whenever MGP is adopted in place of MCP, since BI does not follow from R_S and LOC_S in this case. Thus a SR interpretation of QP preserving locality is not forbidden [in particular, locality can be recovered inside the SR interpretation proposed elsewhere by the author; see Garola (1991)].

I would like to stress again that the adoption of MGP is consistent with the operational attitude of QP. In addition, MGP also fits in with and integrates some results regarding the incompleteness of QP (Garola, 1992a) obtained in the framework of an epistemological perspective (Garola, 1992b,c) according to which the laws of physics only aim to allow the prediction of truth values of testable sentences whenever the truth values of other testable sentences are known, disregarding sentences that are not testable (though untestable sentences can legitimately occur in the language of physics and can be thought of as having a truth value). Indeed, MGP takes into due account the fact that the deduction of unknown truth values from known ones requires that the latter be *conjointly* known, and frees physics from any engagement regarding the truth values of sentences expressing physical laws under uncontrollable physical conditions (I recall that a sentence of this kind has a truth value in any case; MCP requires that this value is always “true”; MGP only requires that it is “true” in empirically testable situations). Thus, ultimately, MGP is induced by accepting without reserve the classical epistemological distinction between truth and epistemic accessibility, *which allows us to recover a nonclassical physical theory (QP) inside a classical rationality framework.*

It is important to observe that MGP does not imply any modification in standard quantum mechanical predictions. Rather, the consequences of MGP regard the interpretation of QP; indeed, MGP allows us, saving SR, to restore (at a merely semantical level) a kind of “ignorance interpretation” of QP, which means that theories more complete than QP are not logically impossible. Hence one can say that MGP satisfies, in some way, Occam’s razor, since it gives the desired results minimizing the changes in the theory.

Finally, it should be noted that MGP may greatly help in clarifying and solving several open problems in the quantum theory of compound systems and in the quantum measurement theory.

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